



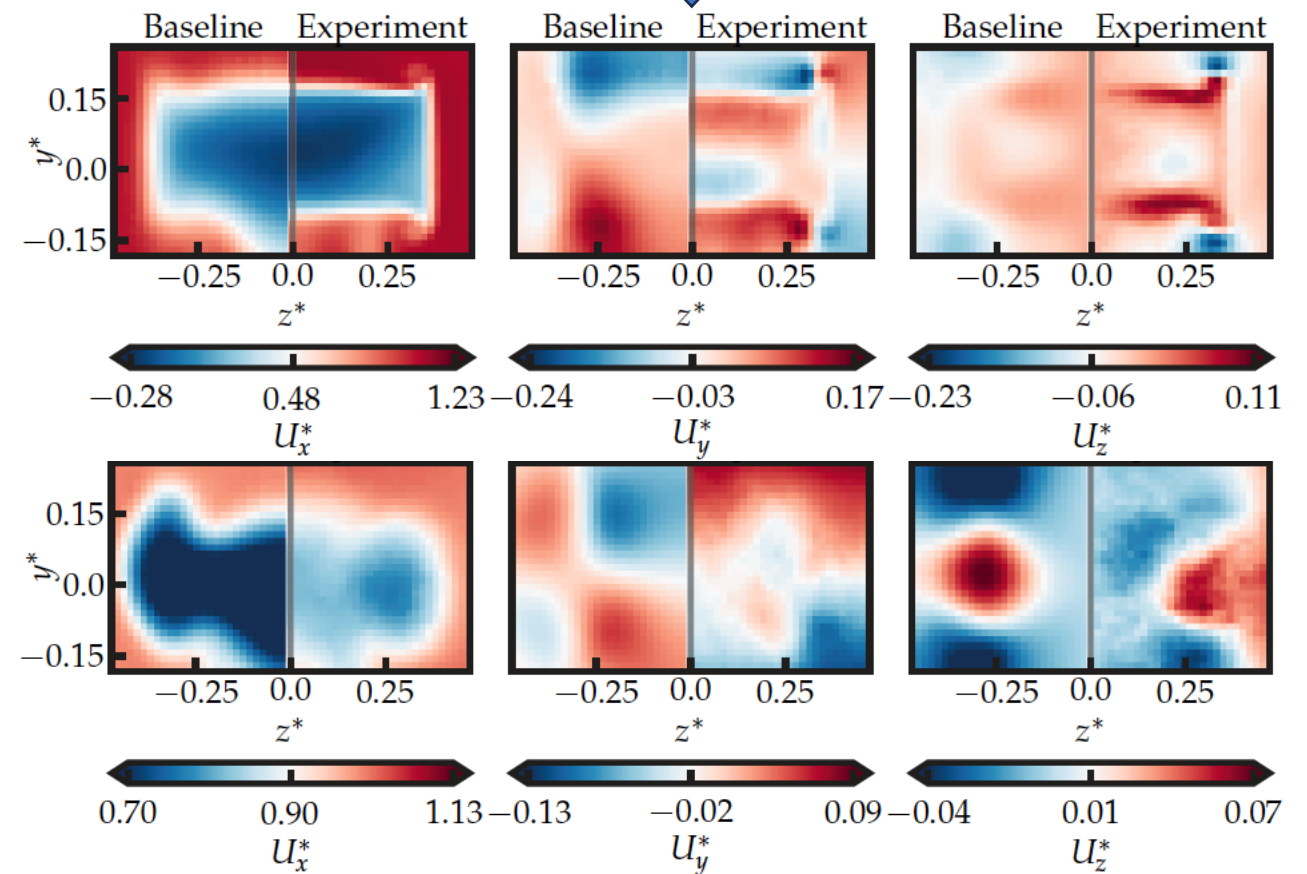
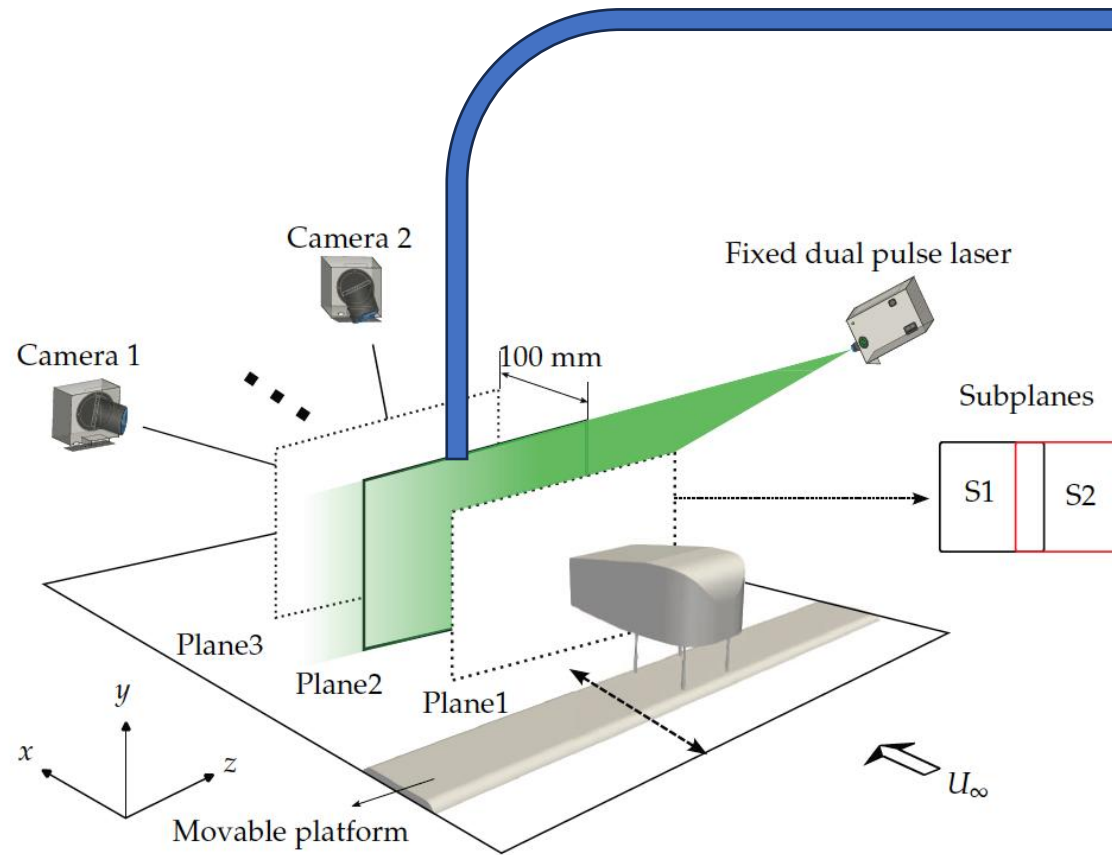
Engineering and
Physical Sciences
Research Council



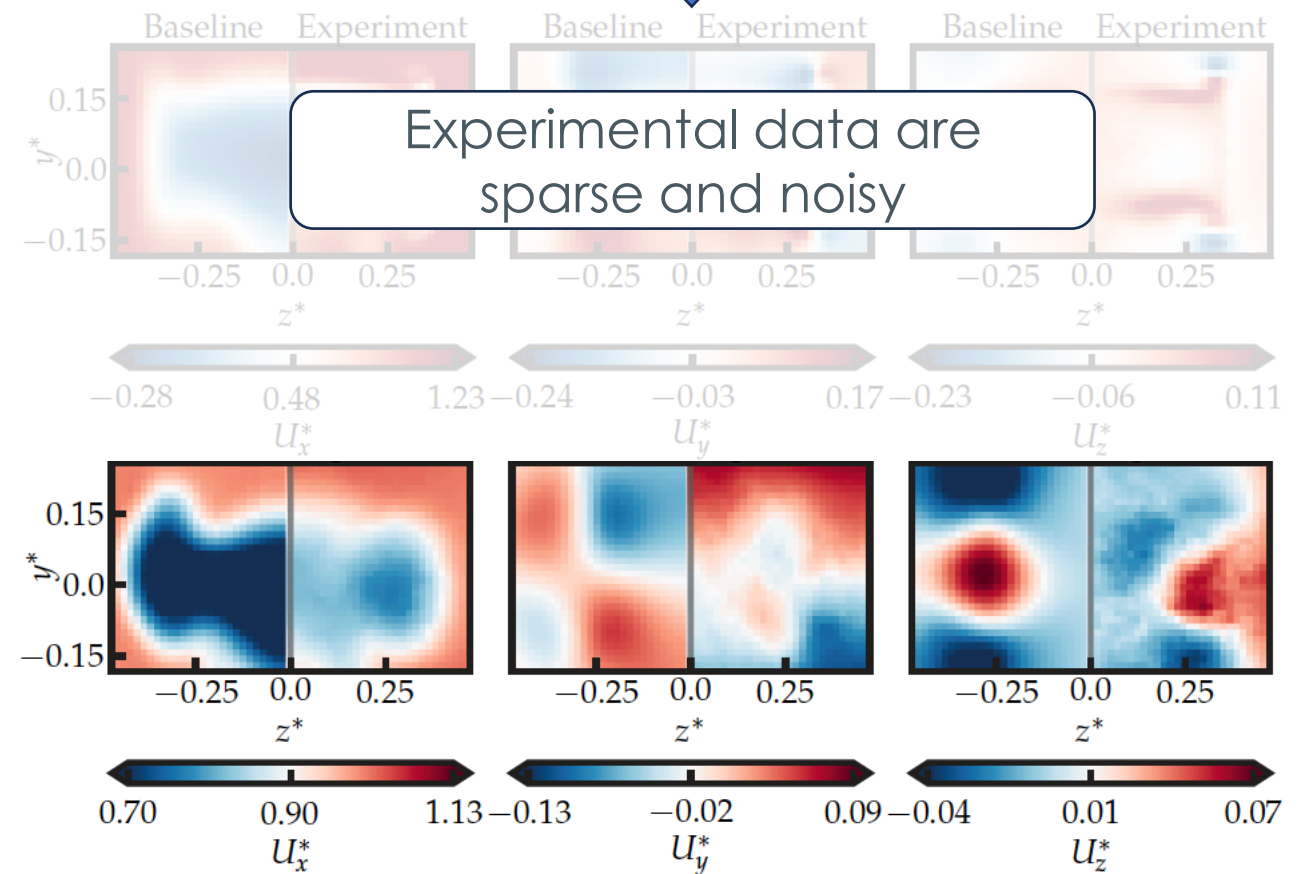
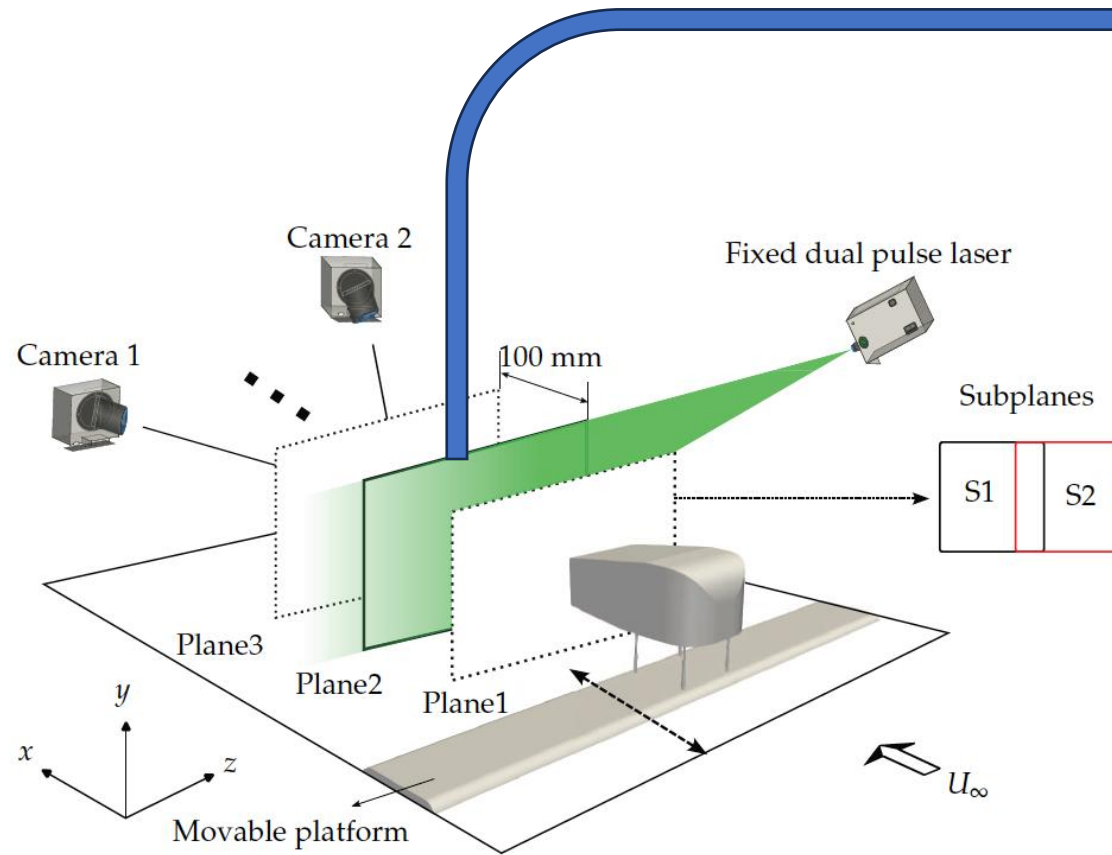
Efficient mean flow reconstruction using sparse experimental data

Uttam Cadambi Padmanaban, Samaresh Midya,
Bharathram Ganapathisubramani, Sean Symon

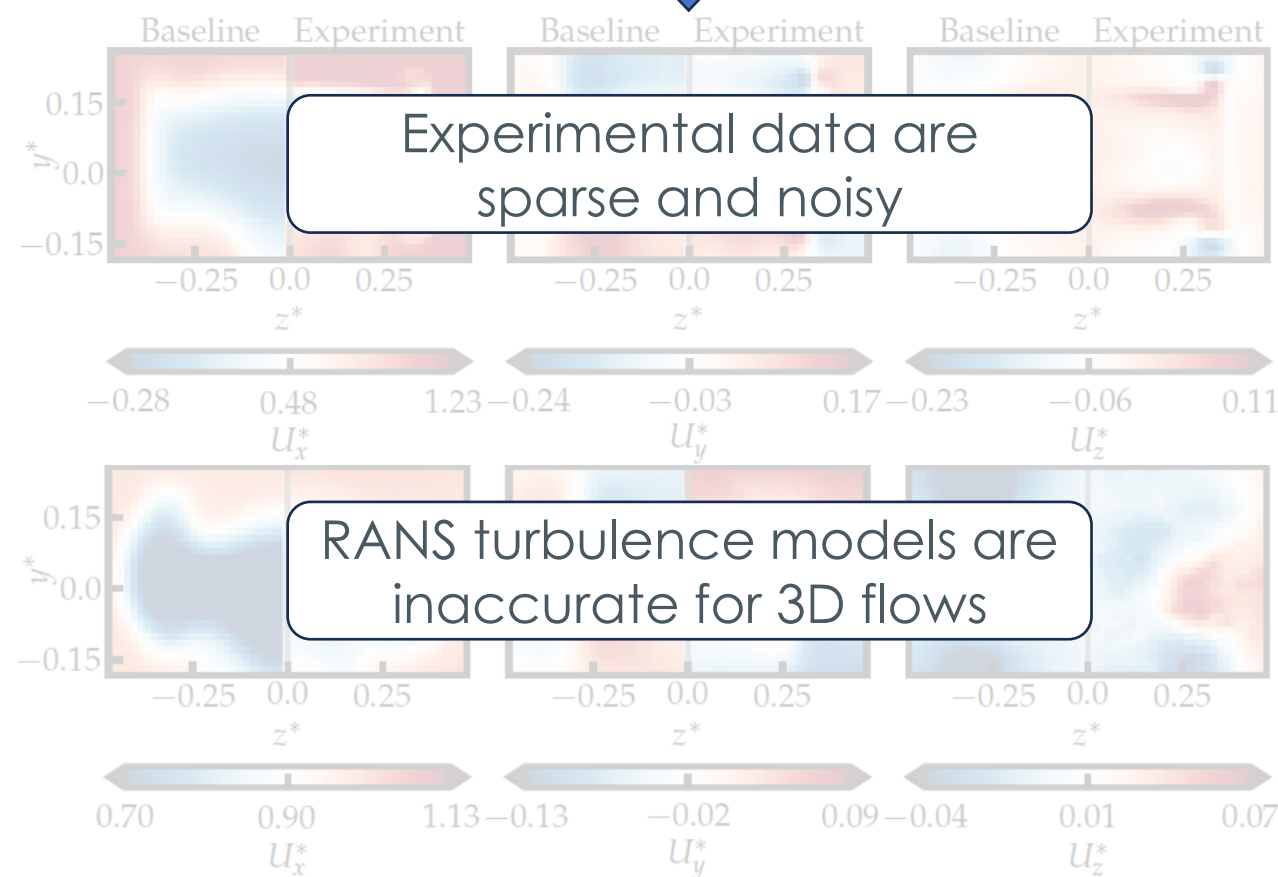
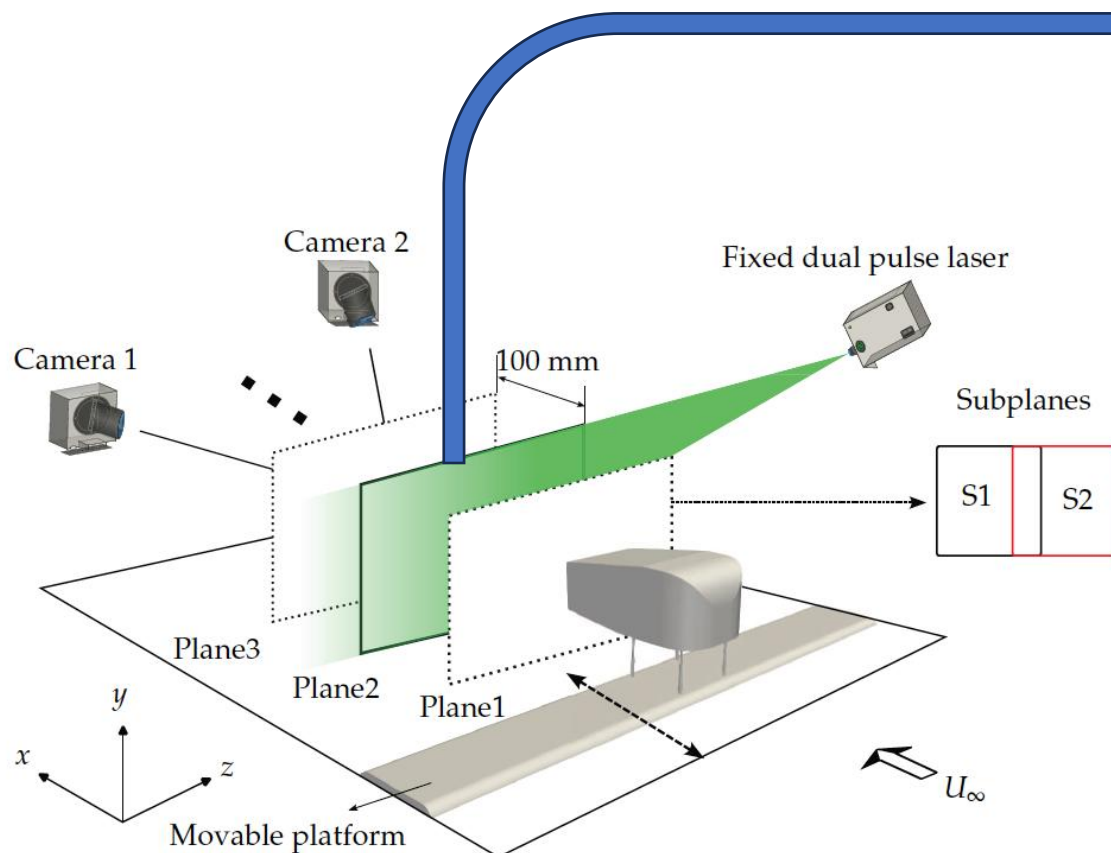
Motivation



Motivation



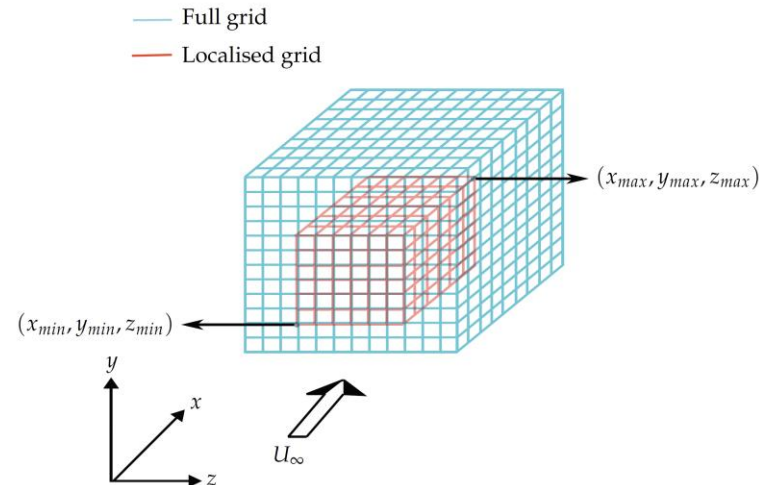
Motivation



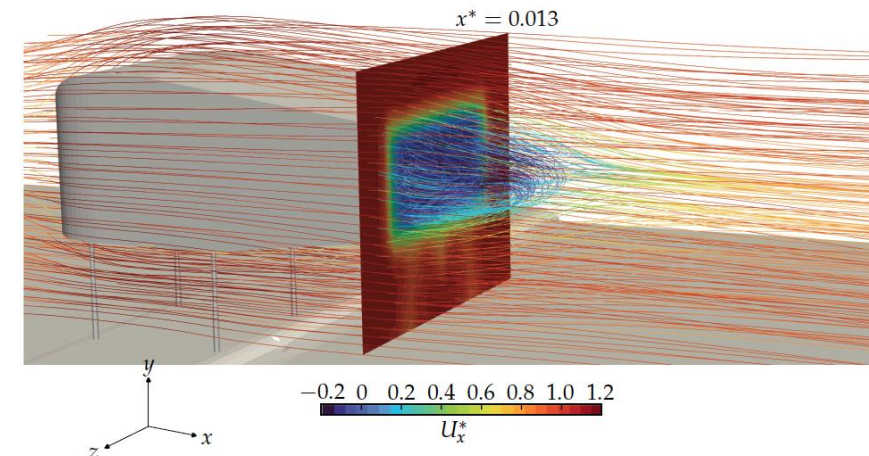
Aims and objectives

Develop efficient methods for assimilating 3D PIV data into RANS turbulence models to enable full-field reconstruction

Objective 1: Develop adjoint localisation for efficient assimilation

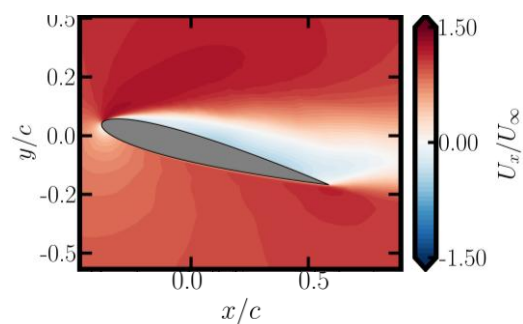


Objective 2: Perform full-field mean flow reconstruction using sparse data



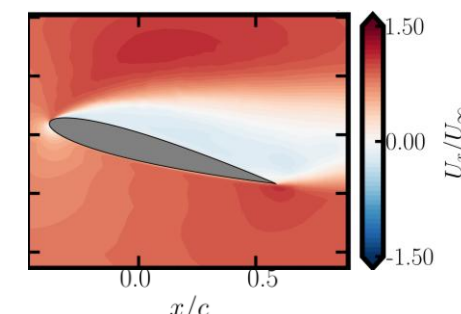
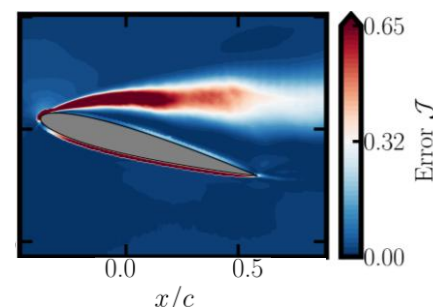
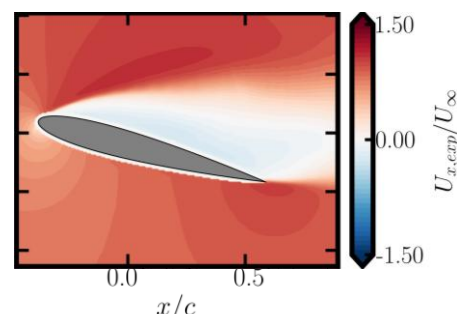
Variational data assimilation

$$\overline{u_j} \frac{\partial \overline{u_i}}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \overline{p}}{\partial x_i} + \frac{\partial (2\nu S_{ij})}{\partial x_j} + f_{Ri} + \boxed{f_{ci}}$$

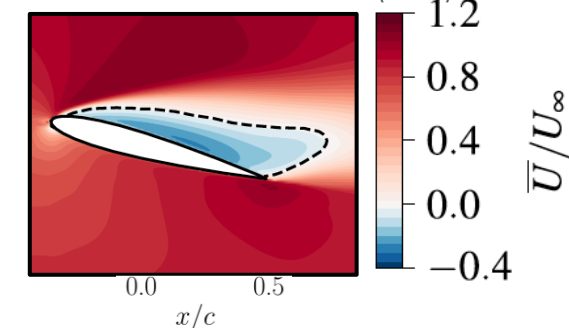


OpenFOAM

Carter, D. W., & Ganapathisubramani, B. (2023)



Cadambi Padmanaban et al. (2026)



Toma et al. (2026) 4

Physics-informed neural network¹

$$\mathcal{L}_{total} = \mathcal{L}_P + \mathcal{L}_B + \mathcal{L}_D$$

¹ Raissi, M., Perdikaris, P., & Karniadakis, G. E. (2019). Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations. Journal of Computational physics, 378, 686-707.

Variational data assimilation

PHYSICAL REVIEW FLUIDS **11**, 014904 (2026)

Three-dimensional variational data assimilation of separated flows using time-averaged experimental data

Uttam Cadambi Padmanaban^{✉,*}, Bharathram Ganapathisubramani^{✉,†} and Sean Symon^{✉,‡}
Department of Aeronautics and Astronautics, *University of Southampton*, Southampton, United Kingdom

 (Received 25 March 2025; accepted 22 December 2025; published 26 January 2026)

We introduce a framework for assimilating planar PIV experimental data using a variational approach to enhance the predictions of the Spalart-Allmaras Reynolds-averaged Navier-Stokes turbulence model. Our method applies three-dimensional constraints to the assimilation of mean velocity data, incorporating a corrective forcing term in the momentum equations. The advantages of this approach are highlighted through a direct comparison with traditional two-dimensional assimilation using the same experimental dataset. We demonstrate its efficacy by assimilating the deep stall flow over a NACA0012 airfoil at a 15° angle of attack and a chord-based Reynolds number of $Re_c \approx 7.5 \times 10^4$. We find that in two-dimensional assimilation, the corrective forcing term compensates not only for physical modeling errors but also for the lack of divergence in the experimental data. This conflation makes it difficult to isolate the effects of measurement inconsistencies from deficiencies in the turbulence model. In contrast, three-dimensional assimilation allows the corrective forcing term to primarily address experimental setup errors while enabling the turbulence model to more accurately capture the flow physics. We establish the benefits of three-dimensional assimilation through its ability to account for the inherent three-dimensionality of the flow, thereby enabling more physically consistent reconstructions of key quantities such as pressure, lift force, and eddy viscosity.

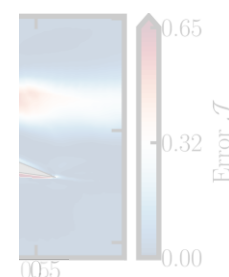
DOI: [10.1103/q7sd-q3dn](https://doi.org/10.1103/q7sd-q3dn)

$$\overline{u_j} \frac{\partial \overline{u_i}}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \overline{p}}{\partial x_i} + \frac{\partial (2\nu S_{ij})}{\partial x_j} + f_{R_i} + \boxed{f_{c_i}}$$

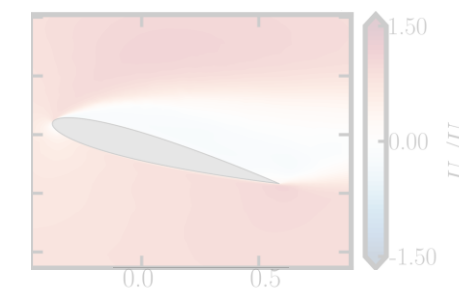
Corrective
forcing

Optimisation
(forcing)

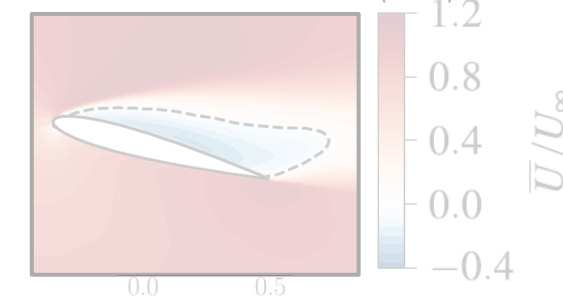
Data
assimilated



$$+ \mathcal{L}_B + \mathcal{L}_D$$



Cadambi Padmanaban et al. (2026)



Toma et al. (2026)

Variational data assimilation

PHYSICAL REVIEW FLUIDS **11**, 014904 (2026)

Three-dimensional variational data assimilation of separated flows using time-averaged experimental data

Uttam Cadambi¹
Department of Aeronautics

and Sean Symon²
Southampton, United Kingdom

6 January 2026)

We introduce a three-dimensional variational data assimilation approach for the Navier-Stokes equations, which assimilates time-averaged experimental data into the flow model. The approach uses a variational formulation of the Navier-Stokes equations, with the experimental data acting as a constraint term in the cost function. The cost function is minimized using an optimization algorithm, resulting in a flow field that is consistent with the experimental data. This approach enables the reconstruction of the flow field, including the flow around a NACA0012 airfoil at a Reynolds number of $Re \approx 7.5 \times 10^4$. The results show that the variational data assimilation approach is able to capture the flow physics accurately, including the flow separation and the formation of a wake. This approach has the potential to be applied to a wide range of flow problems, enabling more accurate and physically consistent reconstructions of flow fields.

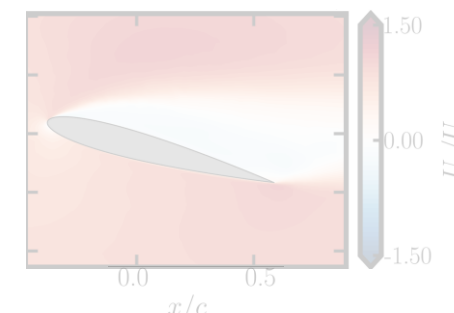
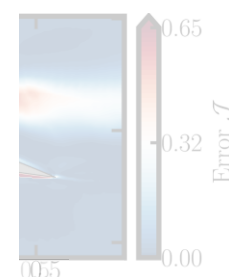
DOI: [10.1103/q7sd-q3dn](https://doi.org/10.1103/q7sd-q3dn)

$$\overline{u_j} \frac{\partial \overline{u_i}}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \overline{p}}{\partial x_i} + \frac{\partial (2\nu S_{ij})}{\partial x_j} + f_{R_i} + \boxed{f_{c_i}}$$

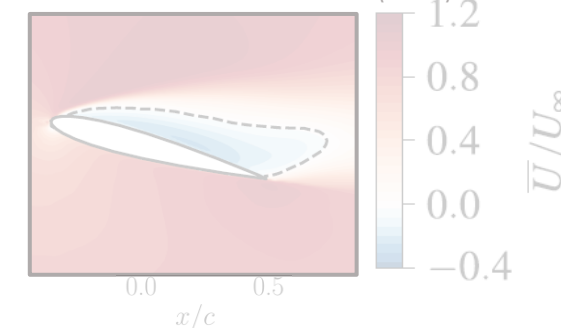
Objective
function

Optimisation
(forcing)

Data
assimilated



Cadambi Padmanaban et al. (2026)



Toma et al. (2026)

4

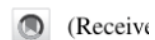
$$+ \mathcal{L}_B + \mathcal{L}_D$$

Variational data assimilation

PHYSICAL REVIEW FLUIDS 11, 014904 (2026)

Three-dimensional variational data assimilation of separated flows using time-averaged experimental data

Uttam Cadambi¹
Department of Aeronautical and Astronautical Engineering, University of Southampton, Southampton, United Kingdom



We introduce a three-dimensional variational data assimilation approach for the Navier–Stokes equations, which assimilates time-averaged experimental momentum and vorticity data into a Reynolds-averaged Navier–Stokes (RANS) simulation. We find that in two dimensions, the assimilation of experimental data into a RANS simulation for physical reconstruction of the flow field is not sufficient. This conflates the deficiencies in the turbulence model to more accurately capture the flow physics. We establish the benefits of three-dimensional assimilation through its ability to account for the inherent three-dimensionality of the flow, thereby enabling more physically consistent reconstructions of key quantities such as pressure, lift force, and eddy viscosity.

DOI: [10.1103/q7sd-q3dn](https://doi.org/10.1103/q7sd-q3dn)



and Sean Symon¹
Department of Aeronautical and Astronautical Engineering, University of Southampton, Southampton, United Kingdom

6 January 2026

ata using a variational data assimilation approach. We find that in two dimensions, the assimilation of experimental data into a RANS simulation for physical reconstruction of the flow field is not sufficient. This conflates the deficiencies in the turbulence model to more accurately capture the flow physics. We establish the benefits of three-dimensional assimilation through its ability to account for the inherent three-dimensionality of the flow, thereby enabling more physically consistent reconstructions of key quantities such as pressure, lift force, and eddy viscosity.

Data-Centric Engineering (2026), 7: e25
doi:10.1017/dce.2026.10054

RESEARCH ARTICLE



Mixed data-source transfer learning for a turbulence model augmented physics-informed neural network

Christian Toma¹, Bharathram Ganapathisubramani and Sean Symon¹

Department of Aeronautical and Astronautical Engineering, University of Southampton, Southampton, UK

Corresponding author: Christian Toma; Email: christian.toma@soton.ac.uk

Received: 12 June 2025 UTC; **Revised:** 17 March 2026 UTC; **Accepted:** 09 May 2026 UTC

Keywords: experimental data assimilation; hard-constraint PINN; physics-informed neural networks; transfer learning

Abstract

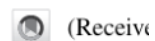
Physics-informed neural networks (PINNs) are a promising alternative for extracting additional time-averaged (mean) flow quantities from experimental data. In the case of particle image velocimetry (PIV), for example, the measured mean flow field is contaminated by noise, has a limited field of view, is restricted to a uniform grid, and does not provide the pressure field. To overcome these limitations, we present a methodology in which PINNs are first trained on a Reynolds-averaged Navier–Stokes (RANS) simulation such that it learns all states at every location in the domain. We then apply transfer learning, which updates the PINN using sub-sampled PIV data. The resulting predictions are in significantly better agreement with the full PIV dataset than PINNs, which are trained on experimental data only. This work builds on the recent literature by integrating a Spalart-Allmaras turbulence model and applying hard constraints to the no-slip wall boundary condition. We apply this methodology to a two-dimensional NACA 0012 airfoil inclined at an angle of attack, $\alpha = 15^\circ$, for two Reynolds numbers of $Re = 10,000$ and $75,000$. The proposed methodology is initially validated using large eddy simulation (LES) data and then demonstrated on experimental PIV data. Our transfer learning approach results in improved predictions and a reduction in training time when compared to using a random network initialization.

Variational data assimilation

PHYSICAL REVIEW FLUIDS 11, 014904 (2026)

Three-dimensional variational data assimilation of separated flows using time-averaged experimental data

Uttam Cadambi¹
Department of Aeronautics



We introduce a variational approach to the assimilation of time-averaged experimental data into a Navier-Stokes turbulence model. We find that in two dimensions, the model can capture the mean flow field, but it fails to capture the pressure field. This conflation of deficiencies is corrected for by the three-dimensional variational data assimilation (3D-VDA) approach. We establish the benefits of three-dimensional assimilation through its ability to account for the inherent three-dimensionality of the flow, thereby enabling more physically consistent reconstructions of key quantities such as pressure, lift force, and eddy viscosity.

DOI: [10.1103/q7sd-q3dn](https://doi.org/10.1103/q7sd-q3dn)

and Sean Symon²
Department of Aeronautics, United Kingdom

6 January 2026)

ata using a variational approach to the assimilation of time-averaged experimental data into a Navier-Stokes turbulence model. We find that in two dimensions, the model can capture the mean flow field, but it fails to capture the pressure field. This conflation of deficiencies is corrected for by the three-dimensional variational data assimilation (3D-VDA) approach. We establish the benefits of three-dimensional assimilation through its ability to account for the inherent three-dimensionality of the flow, thereby enabling more physically consistent reconstructions of key quantities such as pressure, lift force, and eddy viscosity.

Data-Centric Engineering (2026), 7: e25
doi:10.1017/dce.2026.10054

RESEARCH ARTICLE

CAMBRIDGE
UNIVERSITY PRESS

Mixed data-source augmented physics model

Christian Toma¹, Bharat

Department of Aeronautics and
Corresponding author: Christian Toma

Received: 12 June 2025 UTC; E

Keywords: experimental data as

Abstract

Physics-informed neural networks (PINNs) are a powerful tool for solving partial differential equations (PDEs) and have been applied to a wide range of problems in fluid dynamics. However, PINNs often struggle to accurately capture the mean flow field in complex flows, particularly in the presence of turbulence. This work builds on the recent literature by integrating a Spalart-Allmaras turbulence model and applying hard constraints to the no-slip wall boundary condition. We apply this methodology to a two-dimensional NACA 0012 airfoil inclined at an angle of attack, $\alpha = 15^\circ$, for two Reynolds numbers of $Re = 10,000$ and $75,000$. The proposed methodology is initially validated using large eddy simulation (LES) data and then demonstrated on experimental PIV data. Our transfer learning approach results in improved predictions and a reduction in training time when compared to using a random network initialization.



ence model

K

transfer learning

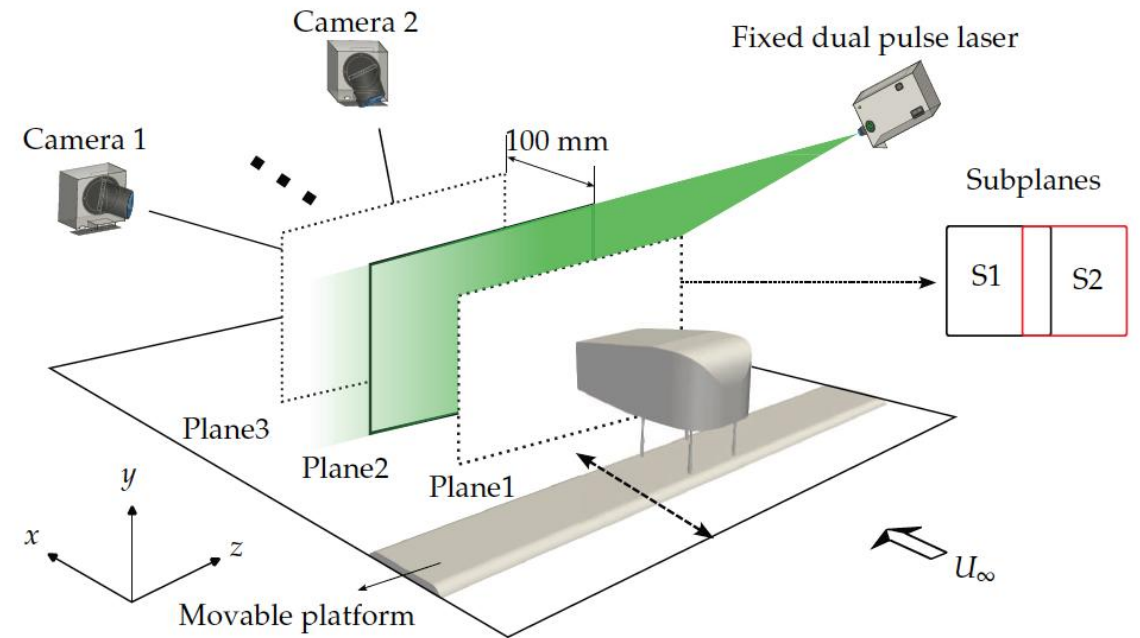
tional time-averaged (V), for example, the uniform grid, and does which PINNs are first t every location in the data. The resulting which are trained on

Experimental details

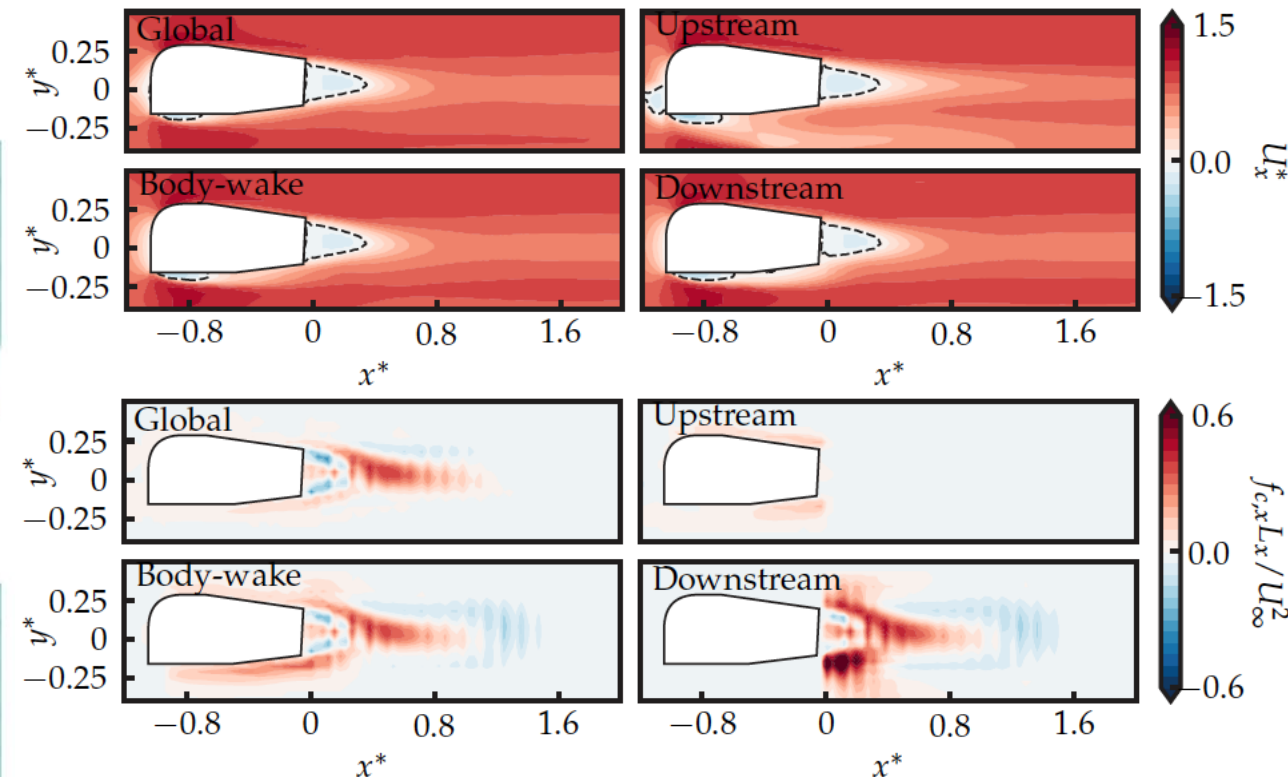
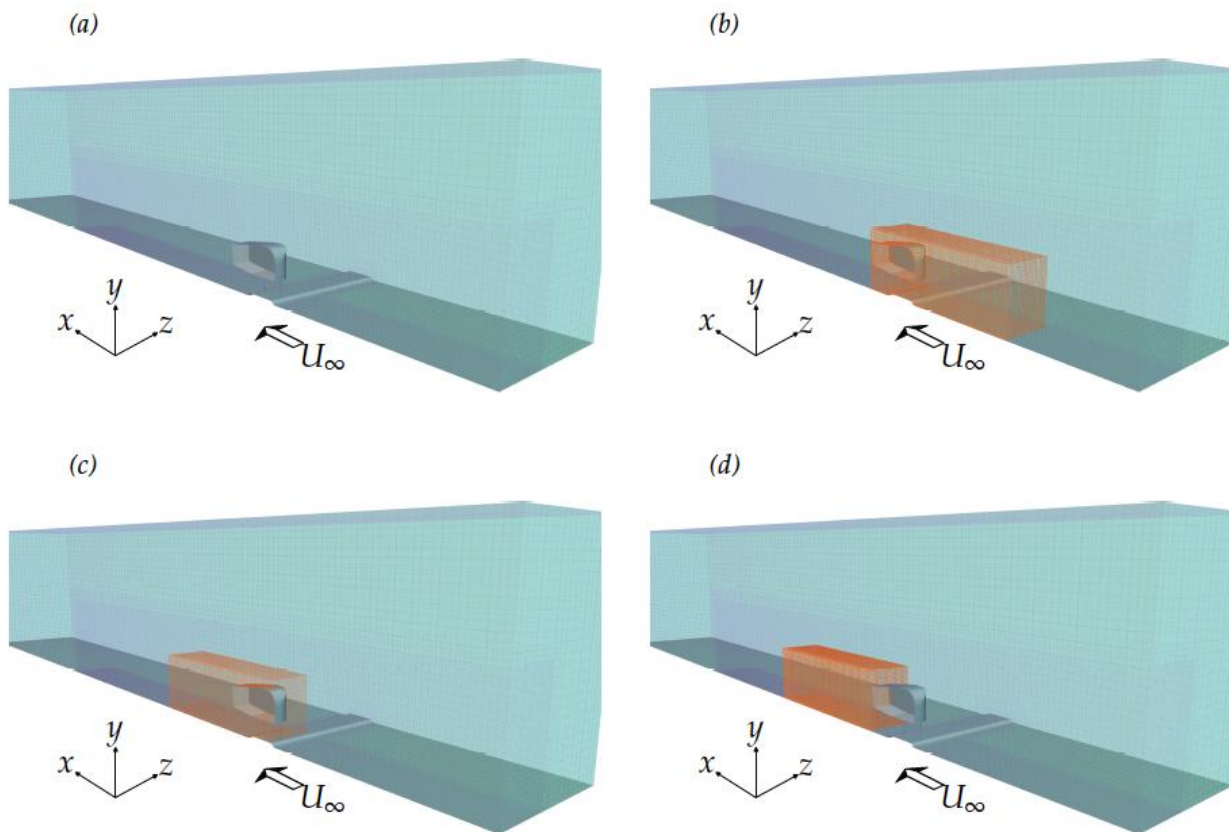
- Data collected in the R. J. Mitchell wind tunnel
- Streamwise length-based Reynolds number = 5.64×10^5



Image courtesy: Ben Purkis



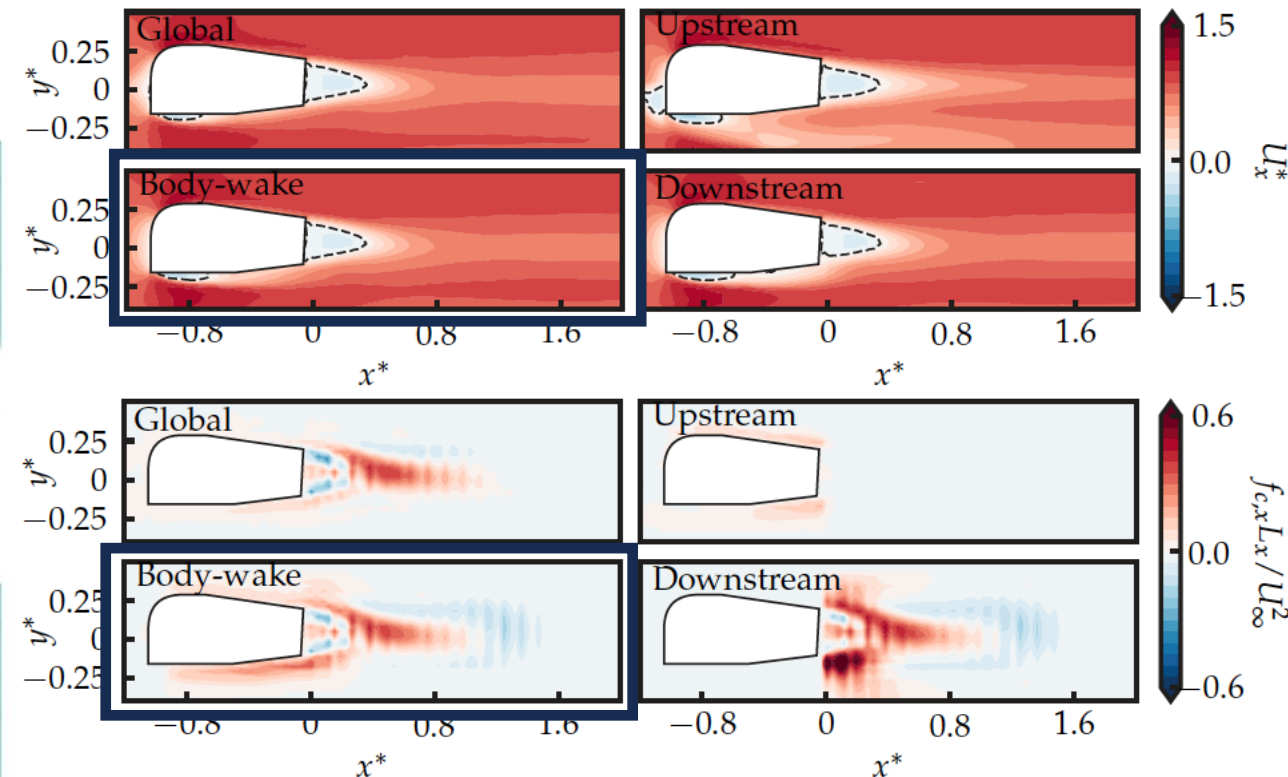
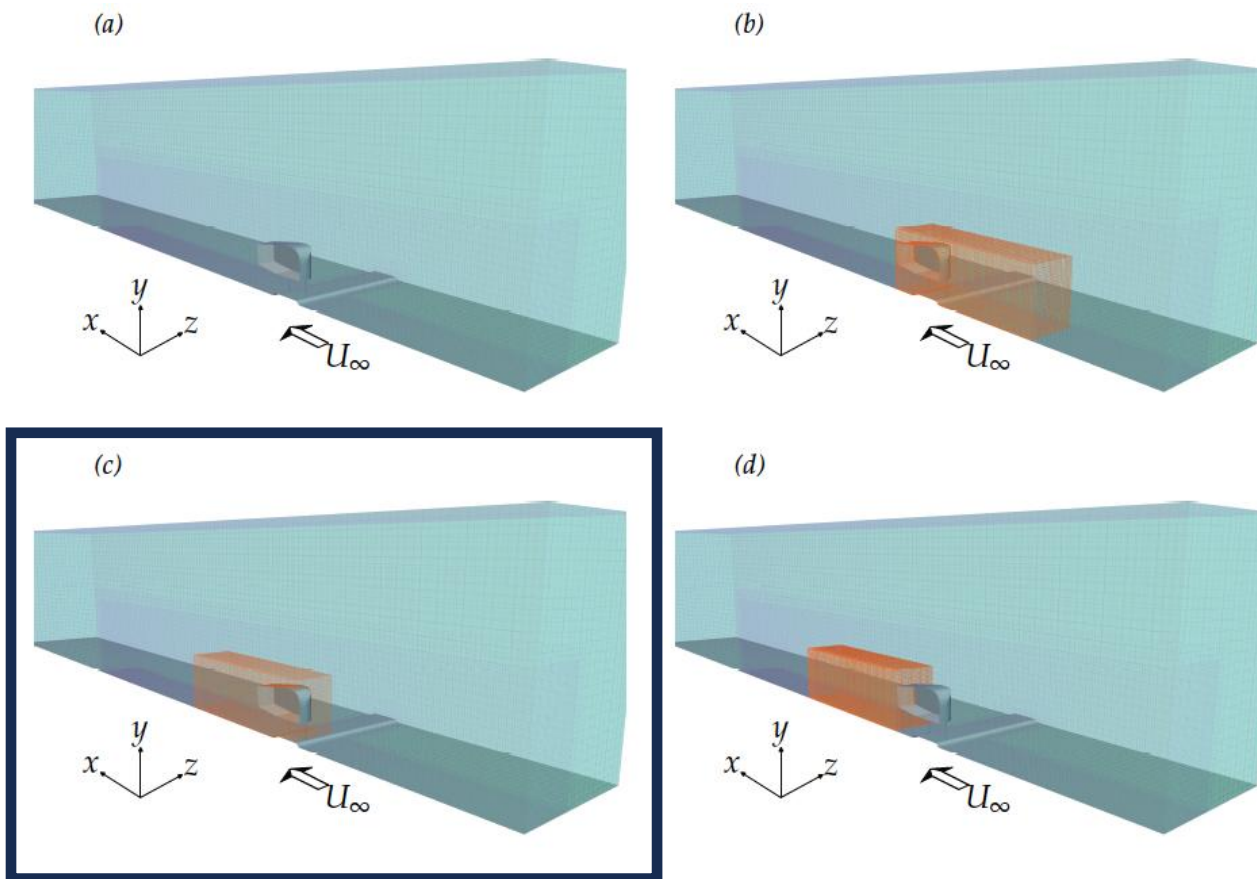
Adjoint localisation



Case	$N_{\Omega_p} / N_{\Omega}$	Peak memory (GB)	$M_{\text{local}} / M_{\text{global}}$
Global	1.000	441.87	1.000
Upstream	0.126	160.18	0.362
Body-wake	0.125	254.89	0.577
Downstream	0.131	227.40 [†]	0.515 [†]

[†] Run on Iridis 6 with 64 active cores

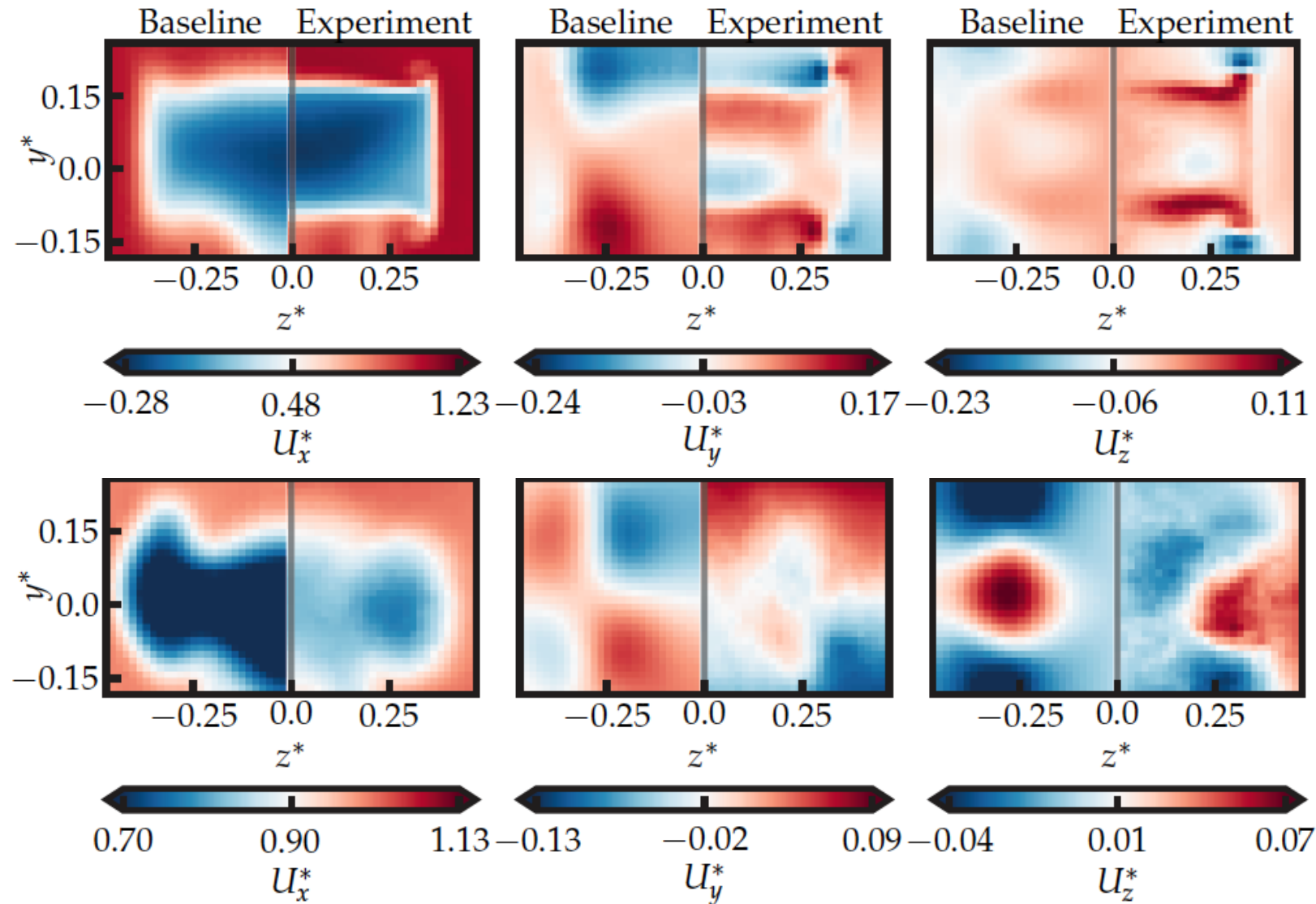
Adjoint localisation



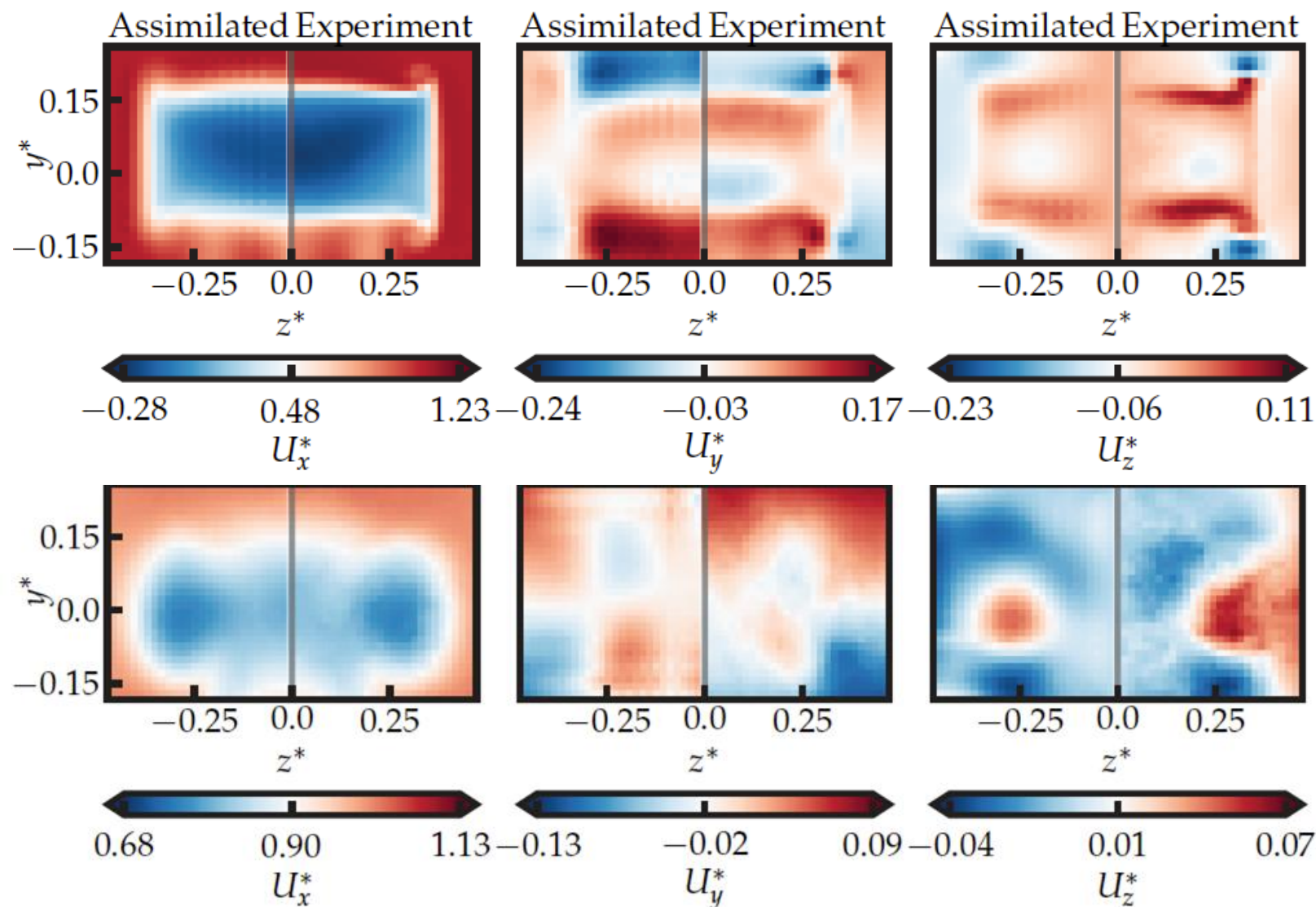
Case	$N_{\Omega_p} / N_{\Omega}$	Peak memory (GB)	$M_{\text{local}} / M_{\text{global}}$
Global	1.000	441.87	1.000
Upstream	0.126	160.18	0.362
Body-wake	0.125	254.89	0.577
Downstream	0.131	227.40 [†]	0.515 [†]

[†] Run on Iridis 6 with 64 active cores

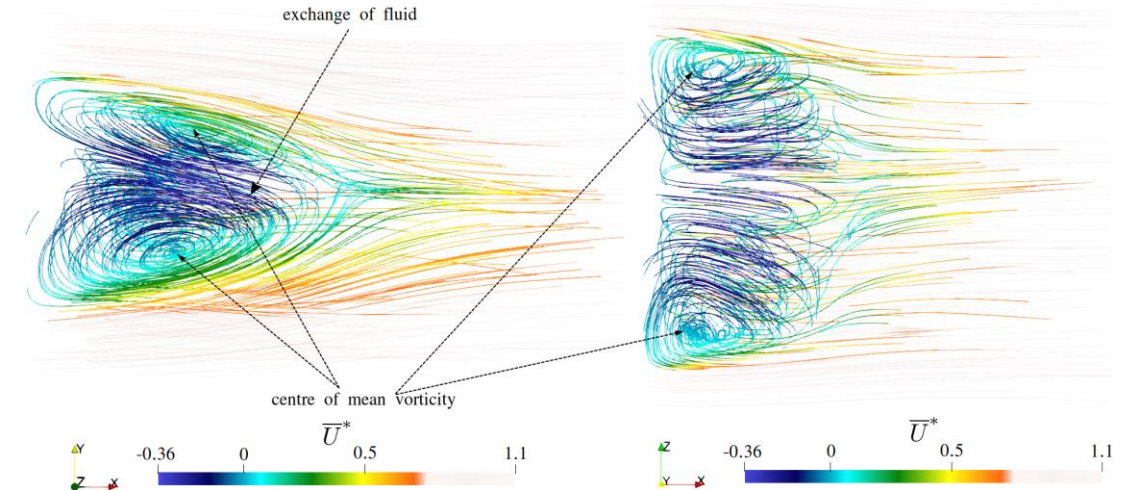
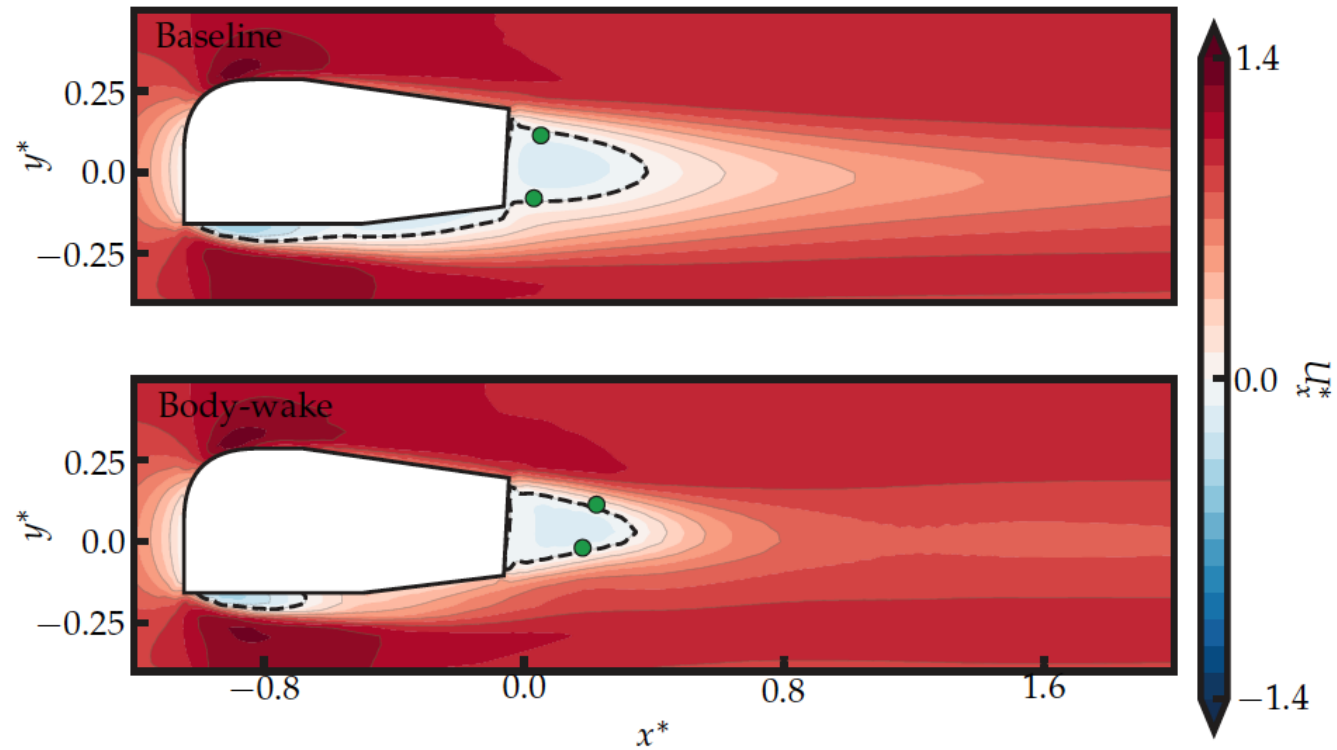
Mean velocity – Planes 2 & 10



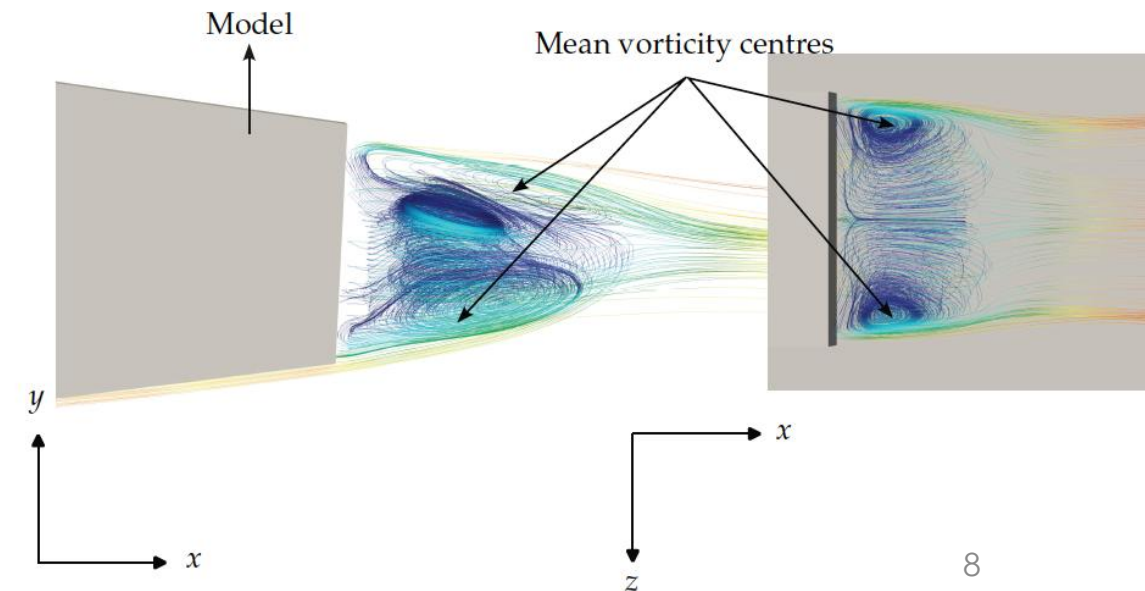
Mean velocity – Planes 2 & 10



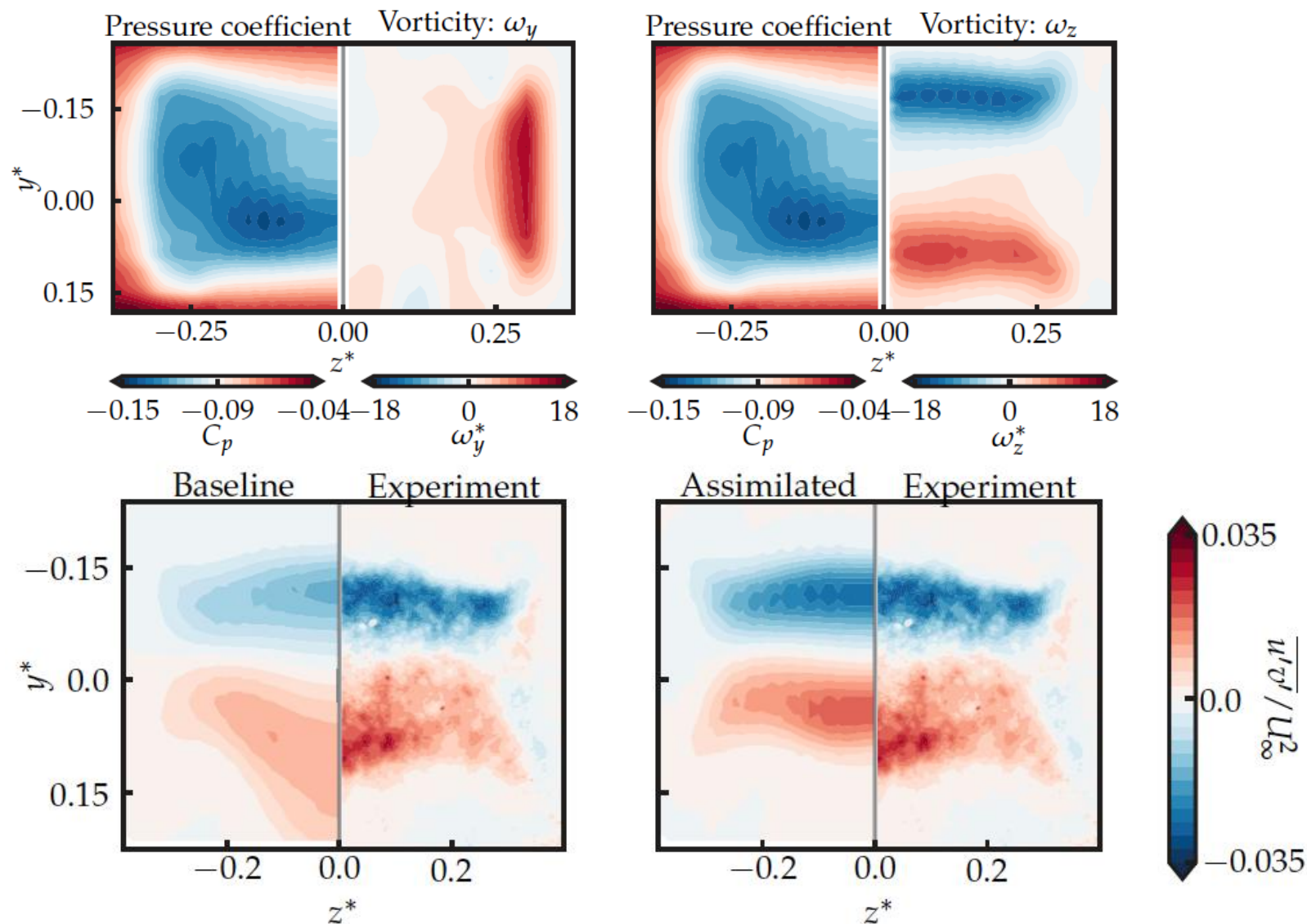
Recirculation bubble



Midya, S. & Symon, S. (2025)



Mean pressure and Reynolds stress



Conclusions and future work

- Conclusions

- Adjoint localization reduces peak memory consumption by at least 43 % without compromising on assimilation quality
- This enables full-field mean flow reconstruction of 3D wake flows using sparse 3D PIV data that captures complex flow physics
- Mean pressure is consistent and Reynolds stresses agree well with the experiment

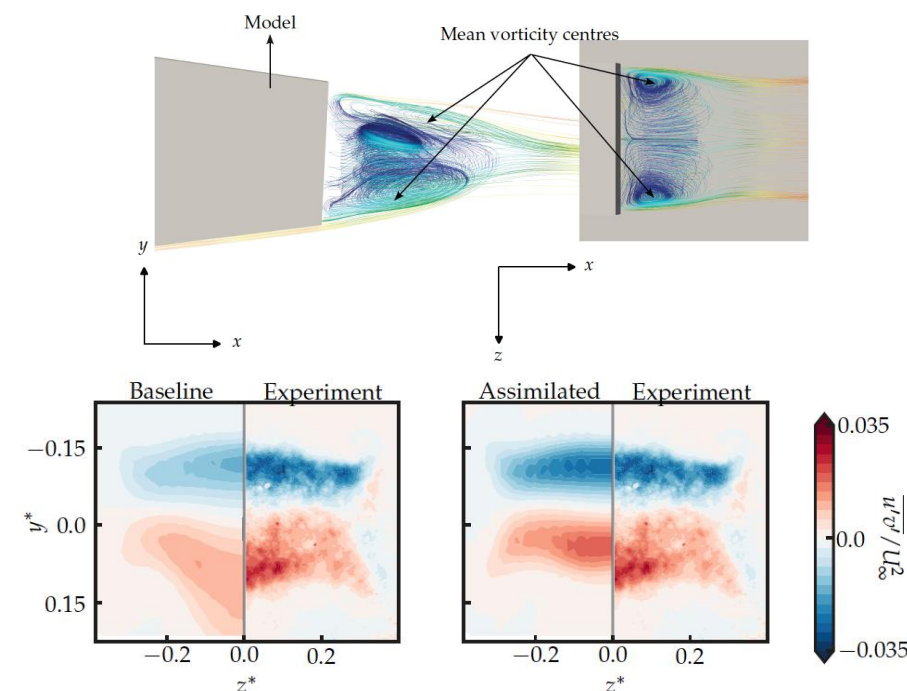
- Future work

- Investigate multigrid approaches for computational cost reduction
- Use mixed data sources for assimilation
- Develop algebraic expressions for the corrections

ucp1y23@soton.ac.uk

Case	$N_{\Omega_p} / N_{\Omega}$	Peak memory (GB)	$M_{\text{local}} / M_{\text{global}}$
Global	1.000	441.87	1.000
Upstream	0.126	160.18	0.362
Body-wake	0.125	254.89	0.577
Downstream	0.131	227.40 [†]	0.515 [†]

[†] Run on Iridis 6 with 64 active cores



Appendix A: Variational data assimilation

- Discrete adjoint method – efficient way of computing gradients.
- Interested in sensitivity – total derivative of the objective function with respect to the design variable.

$$\underbrace{\frac{df}{d\mathbf{x}}}_{1 \times n_x} = \underbrace{\frac{\partial f}{\partial \mathbf{x}}}_{1 \times n_x} + \underbrace{\frac{\partial f}{\partial \mathbf{w}}}_{1 \times n_w} \underbrace{\frac{d\mathbf{w}}{d\mathbf{x}}}_{n_w \times n_x} \longrightarrow \text{Expensive computation}$$

- Finite difference? Good luck, computationally expensive.

$$\underbrace{\frac{\partial \mathbf{R}^T}{\partial \mathbf{w}}}_{n_w \times n_w} \underbrace{\psi}_{n_w \times 1} = \underbrace{\frac{\partial f^T}{\partial \mathbf{w}}}_{n_w \times 1} \quad \frac{df}{d\mathbf{x}} = \frac{\partial f}{\partial \mathbf{x}} - \underbrace{\psi^T}_{1 \times n_w} \frac{\partial \mathbf{R}}{\partial \mathbf{x}}$$

- Take back – need a linear solver for obtaining the adjoint vector. Accurate computation of sensitivity important for optimization.

Appendix B: Derivation of the adjoint

$$\underbrace{\frac{df}{dx}}_{1 \times n_x} = \underbrace{\frac{\partial f}{\partial \mathbf{x}}}_{1 \times n_x} + \underbrace{\frac{\partial f}{\partial \mathbf{w}}}_{1 \times n_w} \underbrace{\frac{d\mathbf{w}}{dx}}_{n_w \times n_x} \quad (1)$$

$$\frac{d\mathbf{R}}{dx} = \frac{\partial \mathbf{R}}{\partial \mathbf{x}} + \frac{\partial \mathbf{R}}{\partial \mathbf{w}} \frac{d\mathbf{w}}{dx} = 0 \implies \underbrace{\left[\frac{\partial \mathbf{R}}{\partial \mathbf{w}} \right]}_{n_w \times n_w} \underbrace{\frac{d\mathbf{w}}{dx}}_{n_w \times n_x} = - \underbrace{\left[\frac{\partial \mathbf{R}}{\partial \mathbf{x}} \right]}_{n_w \times n_x}. \quad (2)$$

$$\frac{d\mathbf{w}}{dx} = - \frac{\partial \mathbf{R}^{-1}}{\partial \mathbf{w}} \frac{\partial \mathbf{R}}{\partial \mathbf{x}}. \quad (3)$$

$$\underbrace{\frac{df}{dx}}_{1 \times n_x} = \underbrace{\frac{\partial f}{\partial \mathbf{x}}}_{1 \times n_x} - \underbrace{\frac{\partial f}{\partial \mathbf{w}}}_{1 \times n_w} \underbrace{\overbrace{\frac{\partial \mathbf{R}^{-1}}{\partial \mathbf{w}}}}_{\psi^T} \underbrace{\frac{\partial \mathbf{R}}{\partial \mathbf{x}}}_{n_w \times n_x}. \quad (4)$$

$$\underbrace{\frac{\partial \mathbf{R}^T}{\partial \mathbf{w}}}_{n_w \times n_w} \underbrace{\psi}_{n_w \times 1} = \underbrace{\frac{\partial f}{\partial \mathbf{w}}}_{n_w \times 1} \quad (5)$$

$$\frac{df}{dx} = \frac{\partial f}{\partial \mathbf{x}} - \psi^T \frac{\partial \mathbf{R}}{\partial \mathbf{x}}. \quad (6)$$